Professor F. Grillot

Problem set 1

EE 270 - Applied Quantum Mechanics

Due Wednesday Oct. 25, 2017 at 8.00 AM

Exercise I (20 points)

(a) Show that de Broglie wavelength of an electron of kinetic energy $E(eV)$ is $\lambda_e = 1.23/\sqrt{E(eV)}$ nm.

(b) Calculate the wavelength and the momentum associated with an electron of kinetic energy 1 eV.

(c) Calculate the wavelength and the momentum associated with a photon that has the same energy.

(d) Show that constructive interference for a monochromatic plane wave of wavelength λ scattering from two plates separated by distance d occurs when $n\lambda = 2d\cos\theta$, where θ is the incident angle of the wave measured from the plane normal and n is an integer. What energy electrons and what photons would you use to observe Bragg scattering peaks when performing electron or photon scattering measurements on a nickel crystal with lattice constant L=0.352 nm and for $n = 1$ and $\theta = 0$?

Exercise II (10 points)

Given that Planck's radiative energy density spectrum for thermal light is

$$U_s(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}\frac{1}{e^{\hbar\omega/k_B T} - 1}$$

show that this expression reduces to the Rayleigh-Jeans spectrum in the low-frequency and to the Wien spectrum in high-frequency. Conclusions.

Exercise III (20 points)
Consider a one-dimensional Gaussian wave packet with an initial state :

$$\psi(x, t=0) = \frac{1}{(2\pi)^{1/4}\sqrt{a}} e^{ip_0x/\hbar} e^{-x^2/4a^2}$$

(a) Calculate the momentum uncertainty at $t = 0$ Δp and the position uncertainty Δx. Show that these obey Heisenberg's uncertainty principle.
(b) Using Fourier analysis, find the distribution of momentum states $\varphi(p)$ used to construct this wave packet.
Hint : A brief look at Gaussian integrals [1]

Exercise IV (10 points)
(a) Show that for a real wave function $\psi(x)$, the expectation value of momentum $< p >$=0. Generalize this result to the case $\psi = c \times \psi_r$, where ψ_r is real and c an arbitrary (real or complex) constant.
(b) Show that if $\psi(x)$ has mean momentum $< p >$, $e^{\pm ik_0x}\psi(x)$ has mean momentum $< p > \pm p_0$ with $p_0 = \hbar k_0$.

Exercise V (20 points)
Show that in momentum space, position is a differential operator, $i\hbar\frac{d}{dp_x}$, by evaluating the expectation value

$$< x >= \int \psi^* \hat{x} \psi dx$$

in terms of $\varphi(p_x)$ defined as the Fourier transform of $\psi(x)$.

Exercise VI (10 points)
A particle of mass m moving in a real potential is described by wave function $\psi(x, t)$ and Schródinger's equation. Show that

$$\frac{d}{dt}\int_{-\infty}^{+\infty} \psi^*(x, t)\psi(x, t)dx = 0$$

so that if the wave function $\psi(x, t)$ is normalized it remains so for all time.

Exercise VII (10 points)
Let the eigenfunctions and eigenvalues of an operator $\hat{A}$ be ϕ_n and a_n,respectively, so that

$$\hat{A}\phi_n = a_n\phi_n$$

1. http://www.weylmann.com/gaussian.pdf

Assume that the function $f(x)$ can be expanded in a Taylor power series expansion. Show that ϕ_n is also an eigenfunction of $f(\hat{A})$ with an eigenvalue $f(a_n)$. That is $f(\hat{A})\phi_n = f(a_n)\phi_n$.